

QUESTION ONE

Let us suppose that there exists a girl. The girl is of a sort often described as quiet, obedient, or agreeable, but not as bright or pretty. She has few friends. Academically, she is almost diligent enough to make up for her lack of talent. She is old enough to remember a time before her brother left for the war, but not yet old enough to understand the gravity of it.

When she has finished her housework for the day, if the weather permits, she goes for a walk—the same route every time—down the track, along the row of poplar trees, around the fields, down to the brook, through the woods, and finally back to the farmhouse. When the mood takes her, she picks wildflowers, or builds dams in the brook, or chases butterflies, or brushes her hand against stinging nettles just to remind herself what it feels like.

She takes pleasure in the indifference of nature. The fact that the birds sing because they want to, rather than simply to impress her, is vital to her for reasons she cannot articulate.

She attends church with her mother every Sunday. She neither actively listens to nor actively ignores the sermons. When her eyes are closed in prayer, the thoughts suddenly drain from her head. She is aware that she is supposed to ask God for something—her brother's wellbeing, presumably?—but no wish feels genuine, and she can't convince herself that anyone can hear her thoughts but her.

She frequently daydreams about the war. First, she imagines herself a nurse, dispensing bandages, sutures, tourniquets, and sympathetic smiles to the hundreds of poor injured souls. Then she imagines herself a soldier, rifle in hand, looking stoically down the trench, tensely waiting for the next offensive. Sometimes she fights alongside her brother. Sometimes she sacrifices her life to save his; other times he saves hers. She does not enjoy daydreaming. When inaccurate it feels disrespectful, and when accurate it feels voyeuristic.

With time, her body shifts and changes beneath her in strange and painful ways. Her mother tells her that this makes her a woman, but she does not feel that to be the case.

The summer days are endless. She tries to read, but the words won't stay in order. She tries to paint watercolours, but the colours all smear together. She tries to keep a diary, but the pages obstinately remain blank. Finally, in wordless despair, she simply sits by the window and watches.

Slowly, gently, like mist rising from the surface of a lake, her mind rises from her body. Her exhaled breath ruffles the waxy leaves of the poplar trees. Birdsong flows from her slightly parted lips, as butterflies chase one another around her eyelashes. The sky throbs rhythmically. She drifts through the air, suspended among the particles of dust and pollen. The girl learns that the vital indifference of nature resides within her as well.

Something deep inside of her is untied, or released, or broken, or lost. She finds that she no longer fears Sunday afternoons, now that she can simply watch, without thought or feeling, the ever-changing view from her window.

Imperceptibly, like the retreating tide, the daylight washes away before her eyes. The shadow of the farmhouse reaches out and gropes further and further into the fields. And the sky—the sky!—has such an aching inhuman beauty as it cycles from full, bold blue, to white, grey, orange, red, purple, and finally to empty, vacant black.

The girl falls out of phase with life. Time slips through her fingers. Entire seasons pass when she isn't looking, just as the sun rises before she wakes. Sometimes she is unnerved by how little this bothers her. But her thoughts and emotions are dispersed so far, and so difficult to find or to hold onto, that she cannot gather the strength to fight it.

Two weeks after the armistice, she sees a man approaching the farmhouse. After only a short hesitation, she sprints from her bedroom, down the stairs, through the kitchen, out of the door, and down the track towards him. Even as her heart pounds and the stones dig into the soles of her bare feet, her mind is shadowed by an awareness that she isn't as happy as she should be.

The girl embraces her brother, burying her face into his chest. She feels a warm wetness against her skin that is not tears. Her brother silently grimaces in pain as blood flows from the newly reopened bullet wound in his abdomen.

We model the flesh comprising the wound as a horizontal cylinder, lying in the x direction, with radius a and length b . Let the pressure at $x = 0$ be given by $p = p_b + p_a \sin \omega t$, where p_b is mean blood pressure, p_a is the amplitude of deviations from p_b , and ω is heartbeat frequency. Let pressure decrease linearly in x , reaching zero at $x = b$. We approximate the blood thickening and wound closing by letting the viscosity increase exponentially with time, such that $\mu = \mu_0 e^{\lambda t}$. We assume incompressible flow, with $Re \ll 1$, and negligible acceleration due to gravity.

- (a) By substituting into the Stokes equations and applying appropriate boundary conditions, show that the velocity field of the blood inside the wound is given by

$$u = \frac{a^2 - r^2}{4b\mu_0 e^{\lambda t}} (p_b + p_a \sin \omega t)$$

Hint: recall that the Laplacian in cylindrical polar coordinates has the form

$$\nabla^2 f = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial f}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial x^2}$$

- (b) Find the volumetric flow rate, defined

$$Q = \iint_A u \, dS$$

where A is the surface of the wound. Plot Q against t .

- (c) Hence find the total volume of blood lost from time $t = 0$ onwards, in terms only of given constants.
- (d) Comment on the validity of the assumptions made in this question.